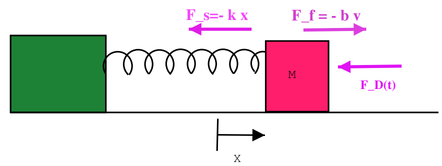
Driven Problem 1 - The Driven Oscillator

A mass M is attached to a spring with force constant k as shown in the figure. The surface upon which the mass slides is frictionless, but the mass is subject to a frictional force $F_f = -bv$ due to air resistance. Initially, the mass is at rest at it's equilibrium point, $x = 0$. At $t = 0$, the mass is subject to a driving force $F_D(t) = \beta t$.



- a) Derive the differential equation that describes the mass' position for $t > 0$.
- b) The solution of this differential equation has two parts, a transient part $x_H(t)$ for which the driving force can be ignored, and a long term solution $x_p(t)$. Assuming that the system is critically damped, write down the soultion for $x_H(t)$ and show that it is a solution of the differntial equation you derived in a) with $F_D(t)$ set to zero.
- c) Solve the differential equation for $x_p(t)$.
- d) Apply appropriate boundary conditions at $t = 0$ and solve for the full motion $x(t) = x_H(t) + x_p(t)$ of the mass for $t > 0$. Make a graph of $xvst$, assuming $k = 3$ N/m and $/beta = 4$N/s.

Driven Problem 2 - Just for Kicks

The same oscillator is instead subject to a periodic "kick" by a driving force given by F$_D(t) = \frac{F_0}{M}$ $nT < t < (n + \frac{1}{2})T$
$= 0$ $(n + frac12)T < t < (n + 1)T$,where T is the time between successive kicks and n is an integer.

- a) Make a graph of $F_D(t)$ vs t. What is its period?
- b) According to Fourier, any periodic function can be written as sum of complex exponentials whose angular frequencies are integer multiples of the frequency of the periodic function, i.e.

$$F_D(t) = \sum_{n=-\infty}^{\infty} F_n e^{in\omega t}, \tag{0}$$

with $F_n = \frac{2\pi}{\omega} \int_0^{\frac{2\pi}{\omega}} dt\, F_D(t)\, e^{in\omega t}$. Calculate the F_n for the periodic kick $F_D(t)$. What frequecy ω should you use?

- c) Solve the driven oscillator equation assuming only one of the terms in the Fourier sum is non-zero.(i.e. Assume $F_D(t) = F_n e^{in\omega t}$ for some n.
- d) Use the priciple of superposition to solve for the long-term part of the oscillator's motion $x_p(t)$. Make a graph of $x_p(t)$ vs t for one period of the "kick".