## Coupled Oscillators

- A Two small masses form are attached to the ceiling to form a compound pendulum. The first mass of mass $M$ is attached directly to the ceiling by a string of length $\ell$ from which it is free to swing. The second mass, of mass $2M$, is attached to the first by a string of length $3\ell$. The strings are massless.
  - a) Draw a picture of this system, denoting the angle between the first string and the vertical by $\theta_1$, and the angle between the second string and the vertical as $\theta_2$.
  - b) In the coordinate system whose origin is at the location where the first string is attached to the ceiling, write the $x$ and $y$ coordinates of each mass in terms of $\theta_1$ and $\theta_2$. Assume that both $\theta_1$ and $\theta_2$ are small so that $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ for both angles.
  - c) Ask the three questions for each mass. Make the same approximations as in part b to obtain the differential equations that govern the behaviour of the system.
  - d) Solve the differential equations in c) by assuming $\theta_1(t) = Re(Ae^{i\omega t})$ and $\theta_2(t) = Re(rAe^{i\omega t})$. What values for $\omega$ and $r$ are required for the solution?
  - e) Initially , the two masses are held so that the top string is vertical and the lower string is at $\theta_2 = \theta_0$. At $t = 0$, the masses are released(from rest). Calculate both angles at time $t$.
- B A particle of charge $q > 0$ and mass $m$ is confined in the x-y plane by a force $\vec{F}_{app} = -m\omega_0^2\vec{r}$, where $\vec{r} = (x, y)$. A magnetic field of strength $B$ is directed in the positive $z$ direction.
  - a) Find expressions for the x and y components of the net force on the particle in terms of its position $\vec{r} = (x, y)$ and velocity $\vec{v} = (v_x, v_y)$.
  - b) Solve the differential equations for the x and y components of the motion.(HINT: Use the same form as the last problem.)
  - c)At $t = 0$, the particle is released from rest at $\vec{r} = (A, 0)$. Find its position as a function of time.
  - d) Use MATLAB or other appropriate software to produce a graph of the charge's trajectory for the case where $qB/m = 2\omega_0$.