Complex Problem 1

The particular solution for the harmonically driven oscillator that we discussed in class was shown to be the real part of

$$x = Ae^{i\omega_D t}. \tag{1}$$

By writing the complex amplitude $A = \frac{F_0/m}{((\omega_0^2 - \omega_D^2) + i\gamma\omega_D)}$ in polar form *before* taking the real part of x, show that the soution for x_p can be written as

$$x_p = B\cos(\omega_D t + \phi_d), \tag{2}$$

where B is the amplitude of the driven oscillator and ϕ_D is the phase shift between the driving force and the position. Derive explicit expressions for B and ϕ_D in terms of the natural frequency ω_0 and damping constant γ of the oscillator.

Complex Problem 2

A driven oscillator only continues to move because the driving force is continually doing work on the system. Recalling that the power(aka the rate of doing work) is $P = \vec{F} \cdot \vec{v}$, calculate the time averaged power input into the system by integrating the instantaneous power over one period of the system.

$$\langle P \rangle = \frac{1}{T}\int_0^T P\,dt \tag{3}$$

Show that the same power is obtained by using complex numbers if we complex conjugate the complex velocity and divide the result by 2, i.e.

$$\langle P \rangle = \frac{1}{2T}\int_0^T F \cdot v^*\,dt. \tag{4}$$

For what frequency is $\langle P \rangle$ maximum?